Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$\therefore\ x=a^2/y$. Let $y=a/n$. $\therefore\ x=an=n^2y$. From (1) and (2),

$$u=\frac{2m}{1+n},\quad v=\frac{2mn}{1+n},\quad u=\frac{v}{n}.$$

Hence, the law: The distance, from the center of the sphere of the point without varies as the square of the distance of the point within from the center of the sphere. The mass of the point without varies directly as the mass of the point within and inversely as the distance from the center.

DIOPHANTINE ANALYSIS.

103. Proposed by **HARRY S. VANDIVER**, Bala, Pa.

Find some solutions of $x^3+ay^3=z^3$ (for x, y, and z) and show that there is an infinite number of solutions corresponding to each integral value of a.

Solution by **L. C. WALKER**, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

For generality we may write

$$x^3+mx^2y+nxy^2+ay^3=x^2\ldots.(1).$$

Let us consider the following products.

$$(u+\alpha v+\alpha^2w)(u+\beta v+\beta^2w)(u+\gamma v+\gamma^2w)\ldots.(2),\text{ and}$$
$$(u'+\alpha v'+\alpha^2w')(u'+\beta v'+\beta^2w')(u'+\gamma v'+\gamma^2w')\ldots.(3),$$

where α, β, γ are the roots of $\theta^3-m\theta^2+n\theta-a=0\ldots.(4)$,

Then $\Sigma\alpha=m$, $\Sigma\alpha\beta=n$, and $\alpha\beta\gamma=a$. Since α is a root of (4), we have

$$\alpha^3=m\alpha^2-n\alpha+a,\text{ or}$$
$$\alpha^4=m\alpha^3-n\alpha^2+a\alpha=(m^2-n)\alpha^2-(mn-a)\alpha+am\ldots.(5).$$

Hence the product of the first factor in (2) by the first factor in (3), is $U+\alpha V+\alpha^2W$, where

$$U=uu'+a(vw'+v'w)+amww',$$
$$V=uv'+u'v-n(vw'+v'w)-(mn-a)ww',$$
$$W=uw'+u'w+vv'+m(vw'+v'w)+(m^2-n)ww',$$

and where the values of α^3 and α^4 were substituted from (5).

In like manner, we find that the product of the six factors in (2) and (3), to be

$$(U+\alpha V+\alpha^2W)(U+\beta V+\beta^2W)(U+\gamma V+\gamma^2W)\ldots.(6).$$

Now put $u'=u$, $v'=v$, $w'=w$; then (6) becomes $(u+\alpha v+\alpha^2w)^2(u+\beta v+\beta^2w)^2\times(u+\gamma v+\gamma^2w)^2$, where

$$U=u^2+2avw+amw^2\ldots.(A),$$
$$V=2uv-2nvw-(mn-a)w^2\ldots.(B),$$
$$W=2uw+v^2+2mvw+(m^2-n)w^2\ldots.(C).$$

Whence we have the general solution of

$$U^3+mU^2V+(m^2-2n)U^2W+nUV^2+(mn-3a)UVW+(n^2-2am)UW^2+aV^3$$
$$+amV^2W+anVW^2+a^2W^3=z^2\ldots.(7).$$

To make (7) applicable to (1), we set $U=x$, $V=y$, $W=0$. Then from (A), (B), and (C), we find

$$u=-\frac{v^2+2mvw+(m^2-n)w^2}{2w},$$
$$x=u^2+2avw+amw^2,$$
$$y=2uv-2nvw-(mn-a)w^2,$$

in which v and w may be chosen at pleasure. When $m=n=0$,

$$u=-\frac{v^2}{2w^2},\quad x=u^2+2avw,\quad y=2uv+aw^2,$$

Hence, since u and v may be chosen at pleasure, there is an infinite number of solutions corresponding to each integral value of a.

Excellent solutions were also received from *G. B. M. ZERR*, and the *PROPOSER*.

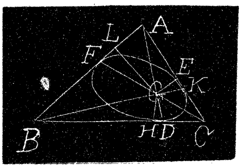
AVERAGE AND PROBABILITY.

126. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Find the average ellipse inscribed in a triangle, so that the sides of the triangle are tangent to the ellipse.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let ABC be the triangle; DEF the inscribed ellipse; D, E, F the points where the sides touch the ellipse; G, the point of concurrence of AD, BE, CF; $GH=l$, $GK=m$, $GL=n$; l, m, n, the perpendiculars from G on the sides; BA, BC, the axes of coördinates; a, b, c the sides of the triangle; $\triangle$, its area; P, the area of the ellipse; and Q, the average area required. Then



$$[(blm+cln)x+(aln+bmn)y-acln]^2-4b^2lm^2nxy=0,$$

is the equation to the ellipse.